The Elusive Fitts’ $D$ and $W$

by
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Abstract

Fitts’ Law estimates the time to acquire a target of width $W$ at distance $D$ is

$$T = a + b \log_2 \left( \frac{D}{W} + k \right)$$

where $a$ and $b$ are respectively the reaction time in seconds and the information transmission rate of the musculoskeletal system in seconds per bit. The constant $k$ exists to better manage extreme cases. The logarithmic component of Fitts’ Law—called the Index of Difficulty by Fitts—quantifies the number of “bits” of information processed during target acquisition. A literature review shows that definitions of $W$ and $D$ for arbitrary convex polygons are inadequately specified. The present paper overcomes this weakness for certain targets by analyzing the case of a cursor positioned at an arbitrary location relative to an arbitrary triangular or convex quadrangular shape designated as the “actual-target.” It is presumed that the user intuits a circular “implicit-target” inscribed within the actual-target that can be hit to acquire the actual-target. It is further presumed the user identifies this implicit-target such that, when the user “hits” the implicit-target by clicking on it, she acquires the actual-target with minimum physical effort. To simplify identification of the width and distance of the optimum implicit-target, the origin of the axis system is translated to coincide with the apex of the actual-target nearest the cursor. The axis system is then rotated to align the X-axis with the bisector of the apex angle. For the initial cursor location and transformed axis system, minimizing calculus is employed to identify the diameter and distance of the implicit-target that minimizes physical effort expended during target acquisition.

Additional analysis determines that for each possible implicit-target there exists a circular locus of points identifying all initial cursor locations for which a given implicit-target is the least effort target. There exists for each apex a maximum circular locus of
points that identifies all initial cursor locations for which the largest implicit-target inscribable within the actual-target is the optimum implicit-target that permits acquisition of the actual-target with least effort. The analysis predicts that any initial cursor location beyond these maximum circular loci will optimally terminate at the center of the largest inscribed circle. If, as in the case of a horizontally positioned rectangle, there are multiple maximum inscribed circles, the closest such circle is sought. This paper shows that ideal terminations of trajectories that are shorter than the radius of the largest loci can be predicted with differing levels of accuracy. This paper also shows that it is impossible to predict the ideal hit point for trajectories commencing on the apex itself or on the extension of the angle bisector.
A Review of Fitts’ Law

Had Fitts based his seminal 1954 article on computer usage, he would have likely posited that a person first expends reaction time comprehending the screen environment, and then formulates a target acquisition plan that identifies the coordinates of the “point-target” within the actual-target that minimizes the effort expended acquiring it. This user then expends physical effort to traverse the mouse toward the point-target. While the point-target is generally missed, it is by an amount of error sufficiently small that the cursor remains within the actual-target. Lastly, the user clicks the mouse and acquires the actual-target. Failure to precisely hit the point-target can be attributed to such human characteristics as a desire for speed, fatigue, muscle tremor, lack of concentration, etc. After analyzing the activities of target acquisition, Fitts concluded the average time required to acquire the actual-target is predicted by:

\[
T = a + b(E) = a + b \times \log_2 \left( \frac{D}{W} + k \right)
\]

where:

- \( a \) = Reaction time (seconds)
- \( b \) = Muscle transfer rate (seconds per bit)
- \( E \) = Index of Difficulty (bits)
- \( W \) = Target width (linear units)
- \( D \) = Distance (linear units)
- \( k \) = Added subsequent to Fitts’ formulation to manage extreme cases

Fitts’ equation, as originally formulated, poorly expressed the relation between \( D \) and \( W \) for extreme cases. Welford (pp 145–148) proposed \( k=0.5 \), justificatied primarily on the expediency that this offers a superior fit to published empirical data. MacKenzie (1989) reviews Shannon’s Theory of Information (which underpins Fitts’ Law), reappraises available empirical data, and concludes the appropriate relation is \( k=1 \). The justification for this decision is that \( k=1 \) provides an even better fit to published empirical data than when \( k=0.5 \). A desirable secondary benefit of MacKenzie’s formulation is that \( E < 1 \) will not occur in instances where the start location is close to the target.
Research has shown Fitts’ Law to be a robust relation that predicts with acceptable accuracy the time it takes to acquire a target. In a general literature review Meyer, et.al., summarize their conclusions of the applicability of Fitts’ Law by stating that “Fitts’ Law appears to reflect a fundamental property of human motor performance” (Fitts, p 341). Studies investigating the computer interface undertaken by Welford, by Jagacinski and Monk, and by Jagcinski, Repperger, Moran, Ward, and Glass show that hand homing times in either direction between keyboard and mouse conform to Fitts’ Law. Experimental work undertaken by Card, English, and Burr, by Jagacinski and Monk, and by MacKenzie (1992) conclude that Fitts’ Law is applicable to mouse-controlled CRT environments under a wide range of conditions. While determination of $k$ is important, MacKenzie observes that studies of Fitts’ Law inadequately address the conceptual problem of providing objective definitions for $D$ and $W$ for many of the target shapes encountered with modern application software. MacKenzie considers the applicability of Fitts’ Law to be questionable in the absence of definitions for Fitts’ $D$ and $W$ appropriate for arbitrarily shaped targets. Gillan, Holden, Adam, Rudisill, and Magee also raise this concern.

Defining the aspect ratio to be $AR = \frac{\text{target length}}{\text{target height}}$, Figure 1 suggests the definitional problems that arise around the values of $D$ and $W$ for targets of different aspect ratios. Parts A through C depict rectangular targets of $AR=1$, $AR>1$, and $AR<1$ respectively. Part D depicts a convex polygon of triangular shape. Researchers generally define the target center to be the intersection of the horizontal and vertical target bisectors. Under prevailing definitions, target distance, $D$, for targets A, B, and C is the length of the dashed lines connecting “+” and the target center. For the square target, A, the width and distance definitions are specific, but characteristics
of the remaining targets do not permit this specificity for either width or distance definitions. The terminus of a traverse to the triangular target until now has been seldom considered, as triangular targets are uncommon to studies involving Fitts’ Law. The analysis presented by this paper will suggest the actual traverses taken by users to be the traverses indicated by the dotted lines.

Validation of Fitts’ Law traditionally entails calculation of Fitts’ Index of Difficulty by directly measuring the target’s dimensions and distance to its center. Fitts created an experimental environment with targets of AR<1 positioned to permit perpendicular traverses. This environment permitted valid direct measurement of $D$ and $W$. While such application software as word processors, spreadsheets, and browsers frequently require acquiring horizontally elongated rectangular targets appearing as text strings, perpendicular traverses to them are frequently not possible. For such targets as these, one can ask several questions. Should length, $L$, or height, $H$, be used for Fitts’ $W$? Are there better criteria for determination of $W$? What alternate criteria exist for measuring $D$? Additionally, given current processing speed and memory capacity, modern Graphics-User-Interfaces employ non-rectangular shapes for targets when such shapes enhance usability. For these more complex targets, what criteria exist for determination of Fitts’ $D$ and $W$?

### Critique of Current Definitions of Fitts’ $D$ and $W$

**Target-Based Analysis of $D$ and $W$**

Early studies investigating Fitts’ Law defined $W$ as a dimension of the target independent of the trajectory performed to acquire that target. This approach will be termed Target-Based Analysis of $D$ and $W$. Table 1 summarizes MacKenzie’s review of the literature for this approach to $D$ and $W$. 

**Table 1**

**Target-Based Definitions of \( W \)**

<table>
<thead>
<tr>
<th>Name</th>
<th>Model</th>
<th>Examples of ( L &amp; H ) giving equal ( E )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Status Quo</td>
<td>( E = \log_2 \left( \frac{D}{H \text{ or } L} \right) )</td>
<td>( (L=9.00, H=0.50) ) ( (L=0.50, H=9.00) )</td>
</tr>
<tr>
<td>Smaller-Of</td>
<td>( E = \log_2 \left( \frac{D}{\min(H, L)} \right) )</td>
<td>( (L=7.00, H=2.00) ) ( (L=2.00, H=4.50) )</td>
</tr>
<tr>
<td>Sum</td>
<td>( E = \log_2 \left( \frac{D}{H + L} \right) )</td>
<td>( (L=8.75, H=0.25) ) ( (L=4.50, H=4.50) )</td>
</tr>
<tr>
<td>Area</td>
<td>( E = \log_2 \left( \frac{D}{H \times L} \right) )</td>
<td>( (L=3.00, H=3.00) ) ( (L=6.00, H=1.50) )</td>
</tr>
</tbody>
</table>

Consider first the implications of MacKenzie’s Status Quo category where either \( H \) or \( L \) is equated to \( W \). Figure 2 illustrates two generic environments a user can face when viewing a rectangular target. The goal is to minimize \( E \) by planning the traverse that reaches the least effort hit-point. The coefficient \( k \) is a constant and not a factor in planning the traverse. Because target-based analysis defines \( W \) solely from target characteristics, \( W \) is independent of \( D \), which means the goal of target-based traverse planning is to minimize \( D \). We presume—and show below—that when facing a target that constrains the trajectory, the computer user seeks to hit a location on the bisector of the constraining boundaries. Figure 2A depicts the case of a cursor initially positioned between extensions of parallel sides. For this case the minimum value for \( D \) is formally stated by:

\[
D = \sqrt{C_y^2 + C_x^2} = \sqrt{C_y^2 + (C_x \tan \theta)^2}
\]
The minimum distance, $D_{min}$, is achieved when $\theta = 0$. Minimizing $E$ through minimizing $D$ is thus not achieved by traversing to the center of the target but rather by traversing perpendicular to the indicated bisector.

Figure 2B reflects a start location that disallows a perpendicular traverse to the closest target boundary. From the preceding argument, $D_{min}$ is achieved by minimizing:

$$D = \sqrt{C^2 + (C \tan(\xi + \theta))^2}$$

Though $D$ is minimized by the traverse having $\theta = 0$, $\xi$ is a constant imposed by the target-cursor environment and disallows a perpendicular traverse to the target. The traverse resulting in the least effort terminates at the target’s edge on the bisector between the top and bottom boundaries. This results in $D_{min} = |Cb|$. The weakness of the target-based analysis is that it makes no allowance for lateral error, which for Figure 2B, results in the need for error correction if any leftward lateral error exists in the initial traverse.

Figure 3 duplicates the environments of Figure 2 except that triangular targets replace the rectangular targets. As with the analysis of rectangular targets, triangular Target-Based definitions do not incorporate aspects of the trajectory into the definition of $W$. Targeting triangular targets for given values for $H$ and $L$ will thus be identical to the acquisition of rectangular targets of equal $H$ and $L$.

Among the definitions of Table 1, we have considered only the Status Quo option, where $W=L$. The preceding analysis applies directly to each interpretation of the “Status Quo” and “Smaller-Of” models, as both $H$
and $L$ are determinate measurements independent of $D$. Consider now, the “Sum” and “Area” models. Since $H$ and $L$ are independent of $D$, both their sum and product are also independent of $D$. Optimum acquisition of the target under the Sum and Area definitions is therefore accomplished by the same traverse strategy that is appropriate for the Status Quo and Smaller-Of definitions.

**Traverse-Based Analysis of $W$**

MacKenzie proposes the Traverse-Based method of defining $D$ and $W$. $W$ is defined to be the length of the line subtended by the target’s boundaries when the traverse is extended through the target. Figure 4 presents the two general environments of a rectangular target for the traverse-based definition of $W$. The dashed lines $\overline{Ct_1}$ and $\overline{Ct_2}$ are bounds that limit the set of traverses that are candidates for potential target acquisition. These are bounds because rotation of $\overline{Ct_1}$ counterclockwise about $C$ decreases $W$ more rapidly than $D$, which results in increased $E$. Similarly, clockwise rotation of $\overline{Ct_2}$ about $C$ also decreases $W$ more rapidly than the decrease in $D$. Given these bounds, the traverses considered for target acquisition will be those traverses that terminate on $\overline{QB}$ between the locations intercepted by lines $\overline{Ct_1}$ and $\overline{Ct_2}$; namely on line $\overline{mn}$.

To determine where on $\overline{mn}$ the rational computer user will terminate the traverse, consider any two arbitrary traverse paths $\overline{Cghi}$ and $\overline{Cbcd}$. The traverse-based
definition of $W$ defines target width for the $\overline{Cbcd}$ traverse to be $W_1 = \|bd\| = \frac{H}{\sin \theta_1}$ and target distance to be $D_1 = \|Cc\| = \frac{C_y}{\sin \theta_1}$. When target acquisition is via the $\overline{Cghi}$ traverse, $W_2 = \|qi\| = \frac{H}{\sin \theta_2}$ and $D_2 = \|Ch\| = \frac{C_y}{\sin \theta_2}$. The effort of acquiring the target using $c$ as the point-target is:

$$E_{cc} = \log_2 \left[ \frac{D_1}{W_1} + k \right] = \log_2 \left[ \frac{C_y \sin \theta_1}{H \sin \theta_1} + k \right] = \log_2 \left[ \frac{C_y}{H} + k \right]$$

$$= \log_2 \left[ \frac{C_y \sin \theta_2}{H \sin \theta_2} + k \right] = \log_2 \left[ \frac{D_2}{W_2} + k \right] = E_{ck}$$

With $c$ and $h$ representing any two arbitrary points, this result holds for all points on the line segment $\overline{mn}$. Thus, under the traverse-based model, the effort to acquire the target is independent of the traverse taken.

Figure 5 provides the basis for appraising the traverse-based definition of $W$ applied to triangular targets. The area surrounding a triangular target may be divided into two regions. Traverses of the first region, represented by Figure 5A, start from locations that permit entry onto the target by crossing either of two sides of the triangle. The other region, represented by Figure 5B, identifies start locations of traverses that can only enter the triangle by crossing a single side.

For any cursor location $C$ of Figure 5A, there exists a line, $\overline{LL}$, parallel to $\overline{KQ}$ that contains the nearest apex, $J$. For any arbitrary point where $g \neq J$ on either of the two target sides the traverse can cross, there exists a line $\overline{MM}$, parallel to $\overline{KQ}$ that contains
g. Line pairs $\overline{LL}$ and $\overline{KQ}$ form a rectangular target with ends located at infinite distance. Line pairs $\overline{MM}$ and $\overline{KQ}$ similarly define a rectangular target. Analysis of the traverse-based definition of $W$ found that for rectangular targets, the physical effort expended to acquire the $\overline{LL} \ || \ \overline{KQ}$ target was the same for any two traverses, such as those exemplified by $\overline{CJd}$ and $\overline{Cgi}$. The same is true for rectangle $\overline{MM} \ || \ \overline{KQ}$. Because $D_{\overline{MM}||\overline{KQ}} > D_{\overline{LL}||\overline{KQ}}$ and $W_{\overline{MM}||\overline{KQ}} < W_{\overline{LL}||\overline{KQ}}$, it follows that $E_{\overline{MM}||\overline{KQ}} > E_{\overline{LL}||\overline{KQ}}$. Acquiring the target via $\overline{CJd}$ requires the physical effort expended to acquire rectangle $\overline{LL} \ || \ \overline{KQ}$ while acquiring the target via $\overline{Cgi}$ requires the physical effort expended to acquire rectangle $\overline{MM} \ || \ \overline{KQ}$. Since $E_{\overline{MM}||\overline{KQ}} > E_{\overline{LL}||\overline{KQ}}$, acquiring the actual-target by a traverse along line $\overline{CJd}$ requires less physical effort than doing so along $\overline{Cgi}$. Since, under traverse-based analysis, $g$ represents any point on sides $\overline{JQ}$ or $\overline{JK}$ other than $J$, the optimum traverse for acquiring a triangular target for the scenario illustrated in Figure 5A is on the extension of the line connecting the cursor and the closest apex of the target.

Figure 5B depicts the case of entry into a triangular target can occur only through a single target side. The same reasoning employed with Figure 5A identifies the optimal traverse as being along the line connecting the initial cursor location and the farthest apex. For Figure 5B, this is a traverse $\overline{Cc}$.

**Footprint-Based Analysis of $D$ and $W$**

The footprint-based model presumes that users first appraise the environment of the “actual-target” to identify an “implicit-target” within the actual-target that minimizes the effort of acquiring the actual-target. It is presumed the computer user understands that traverse speed, fatigue, arm tremor, etc. will result in hits occurring in a circular error pattern centered on a “point-target.” The point-target is the hit location that is perceived to minimize the effort of acquiring the actual-target. The goal of footprint-based analysis...
is to identify the implicit-target, *ex ante*, from any specified polygonal actual-target and an arbitrary cursor location.

**Principles of Target Constraint**
Define the criterion boundary to be the boundary of the actual-target that is crossed during acquisition of the implicit-target. Additionally, there are two types of boundaries that, when crossed, indicate a traverse error. The first of these is the traverse boundary, which is the boundary crossed if the traverse is linearly extended through the actual-target. Secondly, there are lateral boundaries, which describe boundaries of the actual-target that are crossed by inadvertent movement lateral to the direction of traverse. Finally, define constrained and unconstrained boundaries. A boundary is considered to be a constraining boundary if its presence influences where the traverse is terminated. A target will be termed unconstrained if only the criterion boundary is considered as the user formulates the target acquisition plan. A triangular or quadrilateral target will be said to have single, double and triple constraint depending on how many boundaries the user considers when formulating the target acquisition plan.

![Diagram of target constraints](image)

Intuitive appraisal suggests Figure 6A is an unconstrained target, such as frequently encountered when editing large graphic objects. Figure 6B suggests a long text string with both ends sufficiently remote that they do not constrain the traverse. With such targets the top boundary is the only constraint influencing where the trajectory will be terminated. Figure 6C might depict a text string approached from the right. Here the traverse and right lateral boundaries constrain the traverse. Figure 6D suggests a toolbar icon with three constraints. Refer to Figure 3 above to intuit constraints for a
triangular actual-target. It is apparent that a single side of a triangular target can be both a lateral and traverse boundary.

Although the preceding logic can be extended to polygonal shapes having more sides, this study confines itself to an appraisal of triangular and quadrangular targets. As will become apparent in the argument to follow, it is relevant to consider how the triangle can be employed to describe the six variations of convex quadrangular targets. The panels of Figure 7 convey that any convex, quadrangular target can be defined by the intersection of two appropriately shaped and positioned triangles, here termed "Generating-Triangles." Any apex of a generating-triangle that is not an apex of the target will be termed an "External-Apex". When two converging sides of a generating-triangle generate an external-apex that apex will be termed a "Finite External-apex". An "Infinite External-apex" is the theoretical intersection at infinite distance of the sides of a generating-triangle. With the external-apex indicated by light dashed lines, Figure 7A shows that two generating scalene triangles of finite height produce the general convex quadrilateral denoted by heavy solid lines. Figure 7B depicts that one generating-triangle of finite height and one of infinite height generates trapezoidal targets. In this case, an isosceles triangle of finite height results in a regular trapezoid. Figures 7C and 7D depict that two generating-triangles of infinite height having non-orthogonal sides produce the parallelogram. Figure 7E depicts two triangles of infinite height with orthogonal sides producing the rectangle sub-class of parallelogram. Figure 7F depicts generating-triangles with orthogonal pairs of parallel sides and equal bases producing the square.
Computer users react to constrained arbitrary polygonal targets at two levels. First, computer users determine the locus of traverse terminations that are considered rational termini of traverses that could acquire the actual-target. Second, users then decide which of these feasible termini has as its footprint the implicit-target that acquires the actual-target with minimum physical effort. We investigate the locus of acceptable termini before turning to the identification of the optimum implicit-target.

To understand how users perceive where traverses into the actual-target should end, consider the logic underpinning Fitts' Law. Fitts argued that $D$ represents the signal being transmitted into the system while $W$ represents noise. The optimum traverse entails acquiring the target in a manner that maximizes the ratio of signal to noise. Shannon’s Information Theory would state this as maximizing the information content of the signal-noise ratio.

Assuming the cursor is within the target, the decision to terminate a traverse controlled by visual feedback can be expressed by answering two questions. First, is the cursor before the point-target? Second, is the cursor beyond this point? To evaluate the information content of this system consider $\overline{WW'}$; i.e., the line segment within the target when the trajectory extends through the target. Consider the point target, $c$, to be a point somewhere on $\overline{WW'}$. When the cursor is within the target, the probability of it being before $c$ is $p_1 = \frac{|Wc|}{|WW'|}$ and the probability of its being after $c$ is $p_2 = \frac{|cW'|}{|WW'|} = (1 - p_1)$. The basis of Shannon’s Information Theory is that the information content, $H$, of $n$ dichotomous decisions of unequal probabilities is:

$$H = \sum_{i=1}^{n} p_i \log_2 \left( \frac{1}{p_i} \right).$$

The information content of the location of $c$ has two possibilities that are expressed as:
\[
H = p_1 \log_2 \left( \frac{1}{p_1} \right) + p_2 \log_2 \left( \frac{1}{p_2} \right) = p_1 \log_2 \left( \frac{1}{p_1} \right) + (1 - p_1) \log_2 \left( \frac{1}{(1 - p_1)} \right)
\]
\[
= -p_1 \log_2 (p_1) - \log_2 (1 - p_1) + p_1 \log_2 (1 - p_1)
\]
\[
= -p_1 \ln (p_1) - \ln (1 - p_1) + p_1 \ln (1 - p_1)
\]
\[
\frac{dH}{dp} = \frac{d}{dp} \left[ -p \ln (p) - \ln (1 - p) + p_1 \ln (1 - p) \right]
\]
\[
= -\frac{p}{\ln 2} \ln p + \frac{p}{\ln 2} \frac{d}{dp} \ln (1 - p) + \frac{1}{\ln 2} \frac{d}{dp} \ln (1 - p) + \frac{p}{\ln 2} \frac{d}{dp} \ln (1 - p) + \frac{p}{\ln 2} \frac{1}{(1 - p)}
\]
\[
= -\frac{1}{\ln 2} \ln p - \frac{1}{\ln 2} + \frac{1}{\ln 2} \frac{1}{(1 - p)} + \frac{1}{\ln 2} \ln (1 - p) - \frac{1}{\ln 2} \frac{p}{(1 - p)}
\]

Canceling terms and setting to zero to find the maximum results in
\[
-\ln p - 1 + \frac{1}{1 - p} + \ln (1 - p) - \frac{p}{1 - p} = 0
\]
\[
-\ln p + \ln (1 - p) = 1 - \frac{1}{1 - p} + \frac{p}{1 - p}
\]
\[
= 1 - p - 1 + p
\]
\[
= 0
\]
\[
\ln (1 - p) = \ln p
\]
\[
(1 - p) = p
\]
\[
p = 0.5
\]

This results in maximum information at \( p = 0.5 \). Thus, when planning acquisition of the actual-target, the user first identifies candidates as being those traverses that terminate on the mid-point between relevant target constraints.
The computer user now decides which particular implicit-target from among this infinite number of implicit-targets minimizes the effort of actual-target acquisition. Figure 8 helps conceptualize how the selection problem appears for an arbitrary quadrangular target. If the cursor is in the vicinity of acute angle $J$ it can be presumed implicit-targets centered along the angle bisector, $JB_J$, will be considered first. Circles centered on points $u$, $v$, and $r$ describe three such implicit-targets. Implicit-targets $u$, and $v$ are constrained by $KJ$ and $GJ$. Implicit-target $r$ is additionally constrained by $GH$, which makes $r$ the largest implicit-target that can be inscribed in the actual-target. Implicit-targets to the left of $r$, such as implicit-target $s$, with centers on bisector $QB_Q$ are constrained by sides $KJ$ and $HG$. For a cursor in the vicinity of obtuse angle $K$, existence of implicit-targets $l$, $m$, and $n$ suggests that the user makes similar decisions for each apex. However, these appraisals are very qualitative. After all, what is meant by “in the vicinity of…”? Equally, what conditions induce the user to initially appraise implicit-targets centered on the $QB_Q$ bisector rather than the bisector of one of the apexes?

In answering these questions users presumably balance physical effort required to achieve an optimum traverse against the additional effort needed to control such factors as a need for speed, muscle tremor, inattention, and fatigue that result in clicking the actual-target at other than the optimal point-target. If these deviations from the optimum point-target occur randomly, it can be presumed computer users will envisage the terminus of the optimum traverse to be the center of the largest circular footprint that can be inscribed within the target commensurate with minimizing the Index of Difficulty. This circular area is what we have thus far been calling the implicit-target. The goal of this study is to identify the center and radius of the implicit-target that can be hit with minimum physical effort.
That users will behave in the manner expected is justified by the whole of economic science, which is predicated on the presumption that individuals conduct themselves to either maximize gain from a given effort or attain a given goal with least effort. This concept has been explicitly applied to how individuals optimize physiological resources in Navon’s seminal study. If the implicit-target can be identified for an arbitrarily shaped target, its distance, \(D_t\), and width, \(W_t\), applied to Fitts’ Index of Difficulty will provide an objective, quantitative measure of the effort necessary to acquire the actual-target.

### Identification of the Least Effort Implicit-Target

The implicit-target that minimizes acquisition effort can be expressed as a relation of the rate at which the traverse lengthens relative to the rate at which the radius of the implicit-target increases as the implicit-target recedes from the apex nearest the cursor location.

Let’s quickly review how the angle subtended by a target apex is determined. Call this angle \(\delta\) of arbitrary apex \(a\) of an arbitrary triangular target. Determine the length of the triangle’s sides through the following equations:

\[
\overline{ij} = \sqrt{(x_i - x_j)^2 - (y_i - y_j)^2} \quad i \neq j = a, b, c
\]  

Eq 2

The Law of Cosines is then employed to determine \(\delta\), thus:

\[
\delta = \cos^{-1}\left(\frac{ab^2 + ac^2 - bc^2}{2 \times ab \times ac}\right)
\]  

Eq 3
Figure 9 presents the environment of an actual-target apex that subtends an arbitrary angle. It is presumed that the axis system is first translated to place the axis origin at \( Q \) and then rotated to make the X-axis coincide with the bisector \( \overline{QB}_Q \) of angle \( \angle JQG \).

The bisector \( \overline{QB}_Q \) and the target side \( \overline{QJ} \) form angle \( \theta \). The cursor, \( C \), located at \((X_C, Y_C)\), is arbitrarily positioned relative to \( Q \). The distance, \( D_o \), is the length of the traverse from \((X_C, Y_C)\) to the optimal acquisition point, \( t \), located at the unknown coordinates \((X_t, Y_t)\). The circle centered at \( t \) on bisector \( \overline{QB}_Q \) has tangencies with the sides forming apex \( Q \) and denotes an implicit-target, identified here by the gray circle. The radius, \( R_t \), of this circle and its distance, \( D_t \), are determined by:

\[
R_t = X_t \sin \theta
\]  
Eq 4

\[
D_t = \sqrt{Y_t^2 + (X_t - X_C)^2}
\]  
Eq 5

To determine the coordinates of the target acquisition point that minimizes the physical effort, express \( E_t \) as:
\[ E_i = F(G(X_i)) = \log_2 \left( \frac{D}{2R_t} + k \right) = \log_2 \left( \frac{\sqrt{Y_c^2 + (X_t - X_c)^2}}{2X_t \sin \theta} + k \right) \]

\( E_i \) is a strictly increasing function since \( \frac{d(E_i)}{dX_t} > 0 \) for all \( t \). The value of \( X_t \) which

minimizes a strictly increasing function of a function, \( F(G(X_t)) \), is the value of \( X_t \) which

minimizes \( G(X_t) \). This means that the value of \( X_t \) which minimizes \( E_i \) is the same value

which minimizes

\[ G(X_t) = \left( \frac{\sqrt{Y_c^2 + (X_t - X_c)^2}}{2X_t \sin \theta} + k \right) \]

Therefore:

\[
\frac{dG(X_t)}{dX_t} = \frac{d}{dX_t} \left( \frac{\sqrt{Y_c^2 + (X_t - X_c)^2}}{2X_t \sin \theta} + k \right) = \frac{d}{dX_t} \left( \frac{\sqrt{Y_c^2 + (X_t - X_c)^2}}{2X_t \sin \theta} \right) + \frac{dk}{dX_t}
\]

\[
2X_t \sin \theta \times d \left( \frac{\sqrt{Y_c^2 + (X_t - X_c)^2}}{dX_t} \right) - \sqrt{Y_c^2 + (X_t - X_c)^2} \times \frac{d(2X_t \sin \theta)}{dX_t}
\]

\[
= \frac{2X_t \sin \theta \times 0.5 \left( \frac{0 + 2 \left( X_t - X_c \left( \frac{1}{X_t} \right) \right)}{\sqrt{Y_c^2 + (X_t - X_c)^2}} \right)}{(2X_t \sin \theta)^2} - \sqrt{Y_c^2 + (X_t - X_c)^2} \times \frac{2X_t \sin \theta}{X_t}
\]

\[
= 0
\]

Since \( X_t \sin \theta \neq 0 \) for any \( X_t > 0 \) and all possible \( \theta \) of convex polygonal targets, the

preceding can be multiplied by \( (2X_t \sin \theta)^2 \) and terms rearranged to obtain:
\[
\frac{dG(X_t)}{dX_t} = 2 \sin \theta \sin \left( X_t - X_c \right) - 2 \sin \theta \sqrt{Y_c^2 + (X_t - X_c)^2} = 0
\]

Canceling terms gives:

\[
\frac{(X_t - X_c)}{X_t} - \sqrt{Y_c^2 + (X_t - X_c)^2} = 0
\]

\[
(X_t - X_c) - \frac{Y_c^2 + (X_t - X_c)^2}{X_t} = 0
\]

\[
X_t^2 - X_t X_c + \left( Y_c^2 + X_t^2 - 2 X_t X_c + X_c^2 \right) = 0
\]

\[
X_t X_c = Y_c^2 + X_c^2
\]

Given the initial cursor location, the coordinates of the center of the implicit circular target of \( r_t = X_t \sin \theta \) that permits the lowest physical effort necessary for target acquisition are:

\[
\text{Coordinates of } t = (X_t, 0) = \left( \frac{Y_c^2 + X_c^2}{X_c}, 0 \right). \quad \text{Eq 6}
\]

To consider which cursor locations have a given inscribed circle as the implicit-target of lowest physical effort, complete the square for \( X_t X_c = Y_c^2 + X_c^2 \):

\[
Y_c^2 + X_c^2 - X_t X_c = 0
\]

\[
Y_c^2 + X_c^2 - X_t X_c + \left( \frac{X_t^2}{4} \right) = \left( \frac{X_t^2}{4} \right)
\]

\[
Y_c^2 + \left( X_c - \frac{X_t}{2} \right)^2 = \frac{X_t^2}{4}
\]
Rearranging terms gives:

\[
(Y_c + 0)^2 + \left(X_c - \frac{X_l}{2}\right)^2 = \left(\frac{X_l}{2}\right)^2
\]

This specifies a circle of radius \(\frac{X_l}{2}\) with center at \(\left(\frac{X_l}{2}, 0\right)\).

In Figure 10, consider the arbitrary circle centered at \(n\) within the arbitrary, scalene, triangular actual-target, \(\triangle QJG\). The location of \(n\) is on the bisector, \(QB_{\perp}\), of apex \(Q\).

This circle has tangents with at least two sides of the triangle. There also exists a locus of points on the circle centered at \(a\) but outside \(\triangle QJG\) comprising a circular arc for which the circle centered at \(n\) will be the implicit-target of least acquisition effort. This arc is termed the "Equi-Target Loci", (ETL). From the preceding results, the radius, \(r_a\), of the ETL for the implicit-target centered at \(n\) is:
\[ r_n = \frac{X_n}{2} \]  
\[ (X_n, Y_n) = \left( \frac{X_n}{2}, 0 \right) \]

There exists a unique \( ETL_n \) for each \( 0 < X_n \leq X_m \). Circular arcs centered at \( a \) and \( q \) exemplify the \( ETLs \) of but two of the infinite number of possible implicit-targets centered on line \( QB \).

To appraise user behavior from different locations on a single \( ETL \), consider two entities arbitrarily located at points \( U \) and \( V \) on \( ETL_m \). Since both locations are on the same \( ETL \), the user will approach the same implicit-target from either starting point to arrive near the optimal point-target, \( , \). This holds true even though unequal levels of physical effort are expended to acquire the actual-target. This is made apparent by noting that traverses from \( U \) and \( V \) entail traverses of length \( D_U \) and \( D_V \) respectively. The implicit-target width of \( 2r_n \) is common to both traverses. Since \( D_U > D_V \) it follows that:

\[ E_U = \log_2 \left( \frac{D_U}{2r_n} \right) > \log_2 \left( \frac{D_V}{2r_n} \right) = E_V \]

Because the above logic applies to each apex of an actual-target, there exists a "Maximum ETL" for each apex. In Figure 10, these \( MaxETLs \) are represented by the arcs centered at \( q \), \( g \), and \( j \) of apexes \( Q \), \( G \), and \( J \) respectively. When combined, they form the inner boundary of the region of cursor locations for which the optimum implicit-target is the circle centered at \( m \). To show that this is so, consider how to identify the implicit-target appropriate to an arbitrary cursor location, \( F \), outside \( ETL_m \). No implicit-target with a radius smaller than the implicit-target centered at \( m \) will be optimum since such targets are only optimum for \( ETL \) interior to \( ETL_m \). Similarly, no implicit-target can be inscribed within the actual-target having a radius greater than the circle centered at
It follows that for any initial cursor position outside the largest ETL of each target apex, the largest inscribed circle of the actual-target will be the implicit-target with the lowest Index of Difficulty.

The maximum ETL of each apex can be combined to form an inner boundary beyond which any cursor location will have the largest inscribed circle as its implicit-target. This envelope will be called the “Maximum-Target Boundary” (MTB). Thus, even though the physical effort from location $F$ is greater than from location $E$, traverses commencing at locations $F$ and $E$ will both aim for the implicit-target centered on $m$.

The coordinates of a target apex identify the only location common to all ETL of that apex. An initial cursor location on a target apex thus places the cursor on the infinite number of ETL that have the apex as their sole common point. Based on the logic of footprint-based analysis, a cursor positioned on a target apex (such as apex Q of Figure 10) and repeatedly traversed into the target will show a random pattern of hits along the line $Qm$. This result logically follows from Fitts Law since $D_a/W_a = D_n/W_n$ for all $a$ and $n$ such that $X_a < X_a < X_n < X_m$. The explanation of this behavior is that the physical effort of acquiring the actual-target is the same for all traverses that commence at $Q$.

**Analysis of Arbitrary Convex Quadrangular Targets**

Figure 11 depicts a convex, quadrangular, actual-target with apexes at $H$, $G$, $J$, and $K$. The origin of the coordinate system coincides with the external-apex of the $QJG$ triangle. In Figure 11 this is apex Q having angle bisector $\overline{QB_Q}$. Define the “Primary-Triangle” as the generating-triangle having its largest inscribed circle also inscribed in the quadrangular target. Specifically, the primary-triangle is that triangle of the two generating-triangles having the largest inscribed circle of least radius. The other generating-triangle will be termed the “Secondary-Triangle”. The actual-target contains two apexes that are also apexes of the primary-triangle, and these will be termed “Base-
Apexes”. The target also contains two apexes that are not apexes of the primary-triangle, and these are termed “Nonbase-Apexes”.

Recall that the slope of a line \( \overline{ab} \) defined by any two points \( a \) and \( b \), on a Cartesian coordinate system is:

\[
S_{ab} = \frac{y_a - y_b}{x_a - x_b}
\]

with length defined by Eq 2.

The equation of the family of parallel lines defined by two points, \( a \) and \( b \), is defined by:

\[
y = x \tan \theta = \left( \frac{y_a - y_b}{x_a - x_b} \right) x
\]

with the specific equation of the line, \( \overline{ab} \), expressed as:

\[
y = y_a + \left( \frac{y_a - y_b}{x_a - x_b} \right)(x - x_a) = y_a + S_{ab}(x - x_a)
\]

Eq 10

To analyze quadrilateral targets it is further necessary to determine the point of intersection of two non-parallel lines. To this end, consider an additional line defined by points \( c \) and \( d \) having slope \( S_{cd} \neq S_{ab} \). The intersection of lines \( \overline{ab} \) and \( \overline{cd} \) is then determined by:

\[
y_c + S_{ab}(x - x_a) = y_c + S_{cd}(x - x_c)
\]

which, after rearranging terms gives:

\[
x = -k(y_a - S_{ab}x_a) + k(y_c - S_{cd}x_c)
\]

Eq 11

where:

\[
k = \frac{1}{S_{ab} - S_{cd}}
\]

Solving for \( y \):

\[
y = y_a + S_{ab} \left[ -k(y_a - S_{ab}x_a) + k(y_c - S_{cd}x_c) \right] - x_a
\]

\[
= \left( 1 - kS_{ab} \right)(y_a - S_{ab}x_a) + kS_{ab} \left( y_c - S_{cd}x_c \right)
\]

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Since:
\[ 1 - \left( \frac{S_{ab}}{S_{ab} - S_{cd}} \right) = -kS_{cd} \]

The result is:
\[ y = -kS_{cd}(y_a - S_{ab}x_a) + kS_{ab}(y_c - S_{cd}x_c) \]  
(Eq12)

The center of the largest circle tangent to three lines intersecting at two points, g and j, is the intersection of the lines that bisect the angles formed at g and j. To find this intersection, assume an arbitrary line \( \overline{hg} \) defined by coordinates \( (x_g, y_g) \) and \( (x_h, y_h) \).

Define \( \alpha \) as the angle between the X-axis and line \( \overline{hg} \):
\[ \alpha = \tan^{-1}\left( \frac{y_h - y_g}{x_h - x_g} \right) \]

Similarly, define \( \beta \) as the angle between the X-axis and a second arbitrary line \( \overline{jg} \), which intersects line \( \overline{hg} \):
\[ \beta = \tan^{-1}\left( \frac{y_j - y_g}{x_j - x_g} \right) \]

Define \( \delta \) to be the angle relative to the x-axis of the line bisecting \( \angle hgj \):
\[ \delta = \left( \frac{\alpha + \beta}{2} \right) \]

Applying Eq 10 to the bisector, \( \overline{QBQ} \), of the angle formed by the lines \( \overline{hg} \) and \( \overline{gj} \) is:
\[ y = y_g + S_{\overline{gQ}}(x - x_g) \]

where:
\[ S_{\overline{gQ}} = \tan \left( \frac{\tan^{-1}\left( \frac{y_h - y_g}{x_h - x_g} \right) + \tan^{-1}\left( \frac{y_j - y_g}{x_j - x_g} \right)}{2} \right) \]  
(Eq 13)
The equation for a second angle bisector may be represented as: \( y = y_j + S_{jB_i}(x - x_j) \).

From Eq 11 and Eq 12, we can express the intersection of bisectors of two adjacent apexes of a polygonal target as:

\[
x = \left( \frac{1}{S_{gB_k} - S_{jB_i}} \right) \left[ (-y_k + S_{gB_k}x_k) + (y_j - S_{jB_i}x_j) \right] \quad \text{Eq 14}
\]

\[
y = \left( \frac{1}{S_{gB_k} - S_{jB_i}} \right) \left[ S_{jB_i}(-y_k + S_{gB_k}x_k) + S_{gB_k}(y_j - S_{jB_i}x_j) \right] \quad \text{Eq 15}
\]

Define the "Largest Inscribed Extreme Circle" to be the largest circle that can be inscribed within the actual-target that has tangency with three sides of the actual-target. To identify this circle in an arbitrary quadrilateral target, apply Eq 13, Eq 14, and Eq 15 to determine the point where the bisectors of the two base-apexes of the primary-triangle intersect. The radius of the largest inscribed extreme circle is the length of the normal line from the intersection of the bisectors to the nearest side of the quadrilateral. This distance is determined by Eq 2, Eq 3, and Eq 4.

Figure 11 illustrates these concepts. The center of the largest inscribed extreme circle, \( m \), is the intersection of the bisectors of \( \angle H G J \) and \( \angle K J G \). The radius of this circle is the length of the normal line from \( m \) to the nearest side of the quadrilateral; in this case assume this side to be \( H G \). This is line \( m t_m \). Eq 4 is used to determine the length of line \( m t_m \); i.e., \( \| m t_m \| = X_m \sin \theta \).

Similarly, define the "Smallest Inscribed Extreme Circle" to be the smallest circle that can be inscribed within the target having tangency with three sides of the target. To identify the smallest inscribed extreme circle of an arbitrary quadrilateral target, apply Eq 13, Eq 14, and Eq 15 to the intersection of the bisectors of the two non-base apexes. The radius of the smallest inscribed extreme circle is the length of the normal line from the intersection of the bisector intersection determined above to the nearest side of the quadrilateral, a distance determined by application of Eq 2, Eq 3, and Eq 4. In Figure
11, the intersection of bisectors $\overline{HB}_n$ and $\overline{KB}_n$ identifies the center, $n$, of the smallest inscribed extreme circle. To prove this, note that in Figure 11 $\overline{r_1n} \perp \overline{JK}$ and $\overline{r_2n} \perp \overline{KH}$ due to tangency conditions and that $\|\overline{r_1n}\| = \|\overline{r_2n}\|$ because they are radii of the same circle. Since it is known that $\angle r_1nK = \cos^{-1}\left(\frac{r_1n}{Kn}\right) = \cos^{-1}\left(\frac{r_2n}{Kn}\right) = \angle r_2nK$, it follows that line $\overline{Kn}$ bisects $\angle JKH$. Thus $\triangle Knr_1 = \triangle Knr_2$ since triangles having three equal angles and a common side are equal. Similar logic shows that point $n$ falls on line $\overline{HB}_n$, the bisector of $\angle KHG$. This proves that the center, $n$, of the smallest inscribed extreme circle falls on the intersection of the bisectors of the external-apex and non-base apexes; namely, $\angle JKH$ and $\angle KHG$ respectively. The length of the radius of the smallest inscribed circle $r_n$ is determined from Eq 4; (i.e., $\|nr_n\| = X_n \sin \theta$). The intersection of the bisector, $\overline{QB}_n$ of angle $\angle JQG$, and the bisector of either of the non-base apexes also determines the center, $n$, of the smallest inscribed extreme circle. We know that $\overline{r_1n} \perp \overline{JQ}$ and $\overline{r_3n} \perp \overline{OG}$ from tangency conditions and that $\|\overline{r_1n}\| = \|\overline{r_3n}\|$ because both are radii of the same circle. With $\overline{Qn}$ common to $\triangle Qr_1n$ and $\triangle Qr_3n$, we have:

$$\angle r_1Qn = \sin^{-1}\left(\frac{r_1n}{Qn}\right) = \sin^{-1}\left(\frac{r_3n}{Qn}\right) = \angle r_3Qn$$

Note that the radius of the smallest inscribed extreme circle equals the radius of the largest inscribed extreme circle when the primary-triangle has an infinite external-apex.
Once the primary-triangle is identified, footprint-based analysis applies solely to this triangle. After the the MaxETL’s are determined relative to the initial cursor position, quadrangular target analysis is identical to triangle target analysis with one exception. This exception arises when the initial cursor location is located on or within the ETL centered on $e$ and outside either of the non-base apexes centered at $k$ of $h$ in Figure 12. This is the portion of the cursor-target environment where the optimum hit target falls in the area defined by $KHQ$; i.e., the truncated portion of the primary generating-triangle.

These results suggest the following sequence for planning the target acquisition trajectory. If the initial cursor location is further from the centers of all three MaxETLs of the primary-triangle than the radii of each MaxETLs, it is located outside the envelope formed by the three MaxETLs. In this case—as shown during analysis of the triangular target—the optimal traverse from this region is to $m$, the center of the largest inscribed circle of the actual-target. Planning a traverse from initial cursor locations within the
envelope formed by the three MaxETL is based upon which ETL represents the closest apex. If the relevant MaxETL is one of the apexes of the actual-target, the axis is transformed to produce the environment of Figure 11 and the procedure developed for apex ETL analysis of apexes applies. Finally, if this analysis does not identify a MaxETL containing the cursor location, the cursor is located within the MaxETL of the external-apex. Traverses from cursor locations within the MaxETL centered on e further than \(\|R_e\|\) from e will terminate within the quadrilateral. Thus, they are analyzed in the manner described for triangular analysis. Initial cursor locations that are closer to e than \(\|R_v\|\) have the smallest inscribed extreme circle as the implicit-target.

**Analysis of Rectangular Targets**

Figure 7E shows a rectangular target defined by two generating-triangles, each having infinite external-apexes and orthogonal sides. The primary-triangle is the generating-triangle with the narrowest base. Application of the apex ETL analysis developed during the analysis of triangular targets determines the optimal traverse when the initial cursor location is within a MaxETL region. For all other locations, the appropriate implicit-targets will be those having tangency with the parallel sides of the primary-triangle. Since these implicit-targets are all inscribed circles of equal diameter, minimization of \(\log_2 \left( \frac{D}{W} + k \right)\) is achieved by minimization of \(D_i\). Thus, if the line segment defined by the centers of the inscribed extreme circles, line \(\overline{nm}\) of Figure 13, can be reached via a normal traverse, then the normal traverse to \(\overline{nm}\) is the optimal traverse. For all other initial cursor locations, the optimum hit location is the center of the nearest inscribed extreme circle.
To elucidate, if the infinitely remote external-apex of Figure 13 is at \((-X_\infty, 0)\), then \(C_R\) denotes the ETL of infinite radius that contains the center of the right-most inscribed extreme circle. From the geometry of the target and the extreme inscribed circle, it is apparent that \(|\overline{am}| = |\overline{mb}|\) and that \(\overline{am} \perp \overline{mb}\). The shape "maJb" thus forms a square.

Lines \(\overline{mJ}\) and \(\overline{ab}\) are diagonals of \(\Box maJb\) and are thus of equal length and intersect at right angles. Since the hypotenuse of right triangle \(\Delta aJb\) is a diagonal of the ETL centered on \(j\), side \(\overline{KJ}\) of the actual-target and the \(\text{MaxETL}\) of the infinitely remote external-apex intersect at point \(a\). This point is also the intersection between side \(\overline{KJ}\) and the \(\text{MaxETL}\) containing points \(m\) and \(J\). If a normal traverse to the line segment between the centers of inscribed extreme circles is possible, it is then impossible for this initial cursor location to be located within the region bounded by the \(\text{MaxETL}\) of an actual-target apex. Parallel arguments hold for all remaining apexes of the actual-target.

Figure 13 depicts a rectangular target of \(\text{AspectRatio}>>1\) having a coordinate system with its origin at the target’s center-of-gravity and X-axis coincident with the target’s horizontal bisector. Consider \(C_L\) and \(C_R\) to be ETLs of the infinitely remote external-apex located to the left and passing through the centers of inscribed extreme circles.
centered at \( n \) and \( m \) respectively. For analysis of rectangular targets, one of the three following procedures is appropriate:

**First Case:** \( (X_{C_y} < X_c) \)

If the cursor location lies within either the MaxETL centered on “\( j \)” or the MaxETL centered on “\( g \)”, translate the cursor and pertinent apex to produce the alignment depicted by Figure 3 above. Determine the optimum hit location using Eq 6. If the cursor location is not within either the MaxETL centered on “\( j \)” or the MaxETL centered on “\( g \)”, select “\( m \)” as the optimal hit location.

**Second Case:** \( (X_{C_x} \leq X_c \leq X_{C_y}) \).

Select point \( (X_c, 0) \) as optimum hit location.

**Third Case:** \( (X_c < X_{C_y}) \)

This procedure is similar to that outlined for the first case. If the cursor location lies within either the MaxETL centered on “\( k \)” or the MaxETL centered on “\( h \)”, translate the cursor and pertinent apex to produce the alignment depicted by Figure 3 above. Determine the optimal hit location using Eq 6. If the cursor location is not within either the MaxETL centered on “\( k \)” or the MaxETL centered on “\( h \)”, select “\( n \)” as the optimum hit location.

It is possible to appraise rectangular targets with \( AR << 1 \) since such a target can be rotated 90° to produce the environment of Figure 13, thus permitting application of the previous analysis.
Conclusions

Human factors professionals recognize two fundamentally different traverse types: the ballistic traverse and the cognitively controlled traverse. A traverse completed in 200 ms or less is termed ballistic since the average human musculoskeletal system requires this much time before cognitive processes can alter current behavior. By contrast, cognitively controlled traverses that exceed 200 ms in duration provide sufficient time for cognitive processes to adjust the traverse trajectory. If $b$ is known, Fitts’ Index of Difficulty can determine the ratio of $D/W$ which will help identify cursor-target environments in which ballistic traverses prevail. Balakrishnan provides empirical results of 0.24 and 0.35 seconds per bit for the $b$ values of wrist and fingers respectively. Solving the Index of difficulty for the $D/W$ ratio gives: $D/W = 2^{0.5b} - k$. If the value for $k$ is set at 1.0 as MacKenzie’s recommends, the initial cursor position of a ballistic traverse falls within the target. If Fitts’ original formulation is employed—which excludes $k$—the upper bounds of $D/W$ ratios during which ballistic traverses prevail are 1.48 and 1.78 for finger and wrist manipulation respectively. In the most liberal interpretation, these values suggest that ballistic traverses occur when the initial cursor location is no further than three-quarters of the target’s width beyond the target’s edge.

Figure 14 presents a right triangle as an actual-target with the circle centered on $m$ depicting the largest implicit-target. The additional colored inscribed targets depict other possible implicit-targets. Dashed circles surrounding the implicit-targets approximate the distance beyond which cognitively directed traverses will occur. Whether the initial cursor location will result in ballistic or cognitively directed traverses will influence the accuracy of any prediction made by footprint-based analysis for some cursor start locations. Since implications of ballistic and cognitively controlled traverses are not currently incorporated into footprint-based analysis, the following comments suggest areas of further investigation.
When the cursor is initially located on a target apex, all \( ETLs \) intercepted during traverse into the target represent equal levels of effort. Once the cursor is within the actual-target there are no factors external to the target that will influence where the mouse click occurs irrespective of whether the traverse is ballistic or cognitively controlled. Consequently, hit locations will be centered on the apex bisector, randomly spread between the apex and \( m \).

When the cursor is initially positioned near an apex of the actual-target, all traverses into the target will be ballistic. From such locations, initially planned traverses cannot be altered once the traverse is launched. Consider initial cursor location \( C \), for which \( c \) is the optimum hit point. \( C \) is within the zone surrounding the optimum implicit-target
centered at $c$. The traverse from $C$ intersects many of the $ETL$s that suggest new optimum implicit-targets. However, the brevity of the traverse disallows cognitively driven adjustment of the initially planned traverse. The cursor-target environment of location $D$ differs only in that the optimum implicit-target is the maximum inscribed target centered at $m$. Although the traverse intersects most $ETL$s of apex $G$, the user does not have time to adjust the traverse to a more optimum implicit-target because the traverse initiates within the ballistic traverse zone of $m$.

Unless the actual-target contains narrow, acute angle apexes—such as apex $Q$ of Figure 14—footprint-based analysis gives robust predictions for cursor-target environments in most locations beyond the $MTB$ of convex actual-targets. Location $E$ is a good example of such a location. From this location the rational user will select $m$ as the least effort point for acquiring the actual-target. From location $E$, the traverse will not intersect any $ETL$ of apex $J$. It but will intersect $ETL$s of apex $Q$ that have optimum hit-points very close to $m$, the optimum hit-point of $MaxETL_Q$. That this small inducement to traverse adjustment is likely to be inconsequential is amplified by observing that the traverse along line $Em$ does not intercept the $ETL_Q$ until entering the ballistic traverse zone around the implicit-target centered on $m$. Unless a traverse from beyond the $MTB$ passes through the $ETL$ of an apex with sufficient traverse time spent within the $ETL$ to cognitively adjust the traverse, footprint-based analysis will give reasonable time estimates of actual-target acquisition.

During the lengthy planned traverse from $F$ to $m$ along line $Fm$ the greatest portion of the traverse is performed within the $ETL$ associated with apex $Q$. Given the length of this traverse, cognitively controlled adjustment is possible since the traverse trajectory intersects $ETL$s that indicate new optimum hit locations, which, irrespective of the extent of actual adjustments, will be to the left of $m$. While evaluating the extent of this behavior quantitatively awaits empirical research, it can be qualitatively stated that the hit will be on the bisector of the $ETL$’s apex to the left of $m$. 

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When the initial cursor location is embedded in the $ETL$s of an apex but positioned sufficiently far from the optimum hit-point that cognitive control of the traverse is possible, the predictive power of footprint analysis declines. As suggested by the three apexes of Figure 14, this cursor-target environment can exist only with an actual-target having apexes with angles that subtend only a few degrees. Consider initial cursor location $A$. At this location the $ETL$ gradient is very dense. Presume it is difficult for a computer user to correctly identify the actual $ETL$ occupied by the cursor. Presume also that the resulting error is randomly distributed left and right of the correct $ETL$. Because the gradient density left of $A$ is greater than to the right, any error on the left intercepts more $ETL$ than does an equivalent error on the right. The result will be that the dispersion of hits resulting from error to the left of $A$—exemplified by $a^+$—will be to the right of the optimum point-hit, $a$, and exceed the dispersion of hits resulting from error to the right of $A$—as exemplified by $a^-$. If the cursor-target environment results in ballistic traverses, it is predicted the mean observed hit will be closer to $m$ than if the $ETL$ hosting the cursor could be correctly identified. If the traverses from $A$ are cognitively controlled, adjustments to the traverse trajectory will be similar to those that start from location $F$. It can then be predicted that for cognitively controlled traverses the bias toward $m$ will be less severe than with ballistic traverses.
References


Jagacinski and Monk, op. cit. [Is this necessary? I thought you only used Op Cit in footnotes?]


MacKenzie, I., “Fitts’ Law as a research and design tool in human-computer interaction”, Human-Computer Interaction, 1992, pp. 91-139


